

## THE HUMAN HEART IS REPRESENTED BY A GRAPH THEORETICAL NETWORK MODEL

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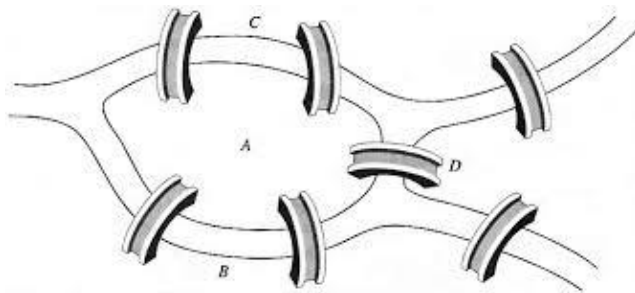
### Abstract

The use of graph theory to the study of the human heart is presented in this paper. In addition to developing a model for the micro-cardiac network system, graph theory techniques are discussed. The major goal is to use network graph theory to analyze the human heart's blood flow system while taking both oxygenated and deoxygenated blood circulation into account. In an electrical circuit represented by a planar graph, the crossing number is found to be an essential parameter for accurate findings. This idea may help us comprehend the blood flow mechanism in the human heart. In addition, a technique to improve the performance of the model is introduced together with the stereographic projection of a graph. This research will definitely be beneficial.

**Keywords:** *image processing; network; graph; circulation; human heart; analysis.*

### Introduction

Graph theory is the study of graphs in the field of mathematics that focuses on the relationship between vertices and edges. It is a well-accepted topic with numerous applications. The Swiss mathematician Leonhard Euler initially presented the fundamentals of graphs in the eighteenth century. Königsberg, a German city, is located on the Pregel River.



The four primary land masses that make up the geographical layout are connected by a total of seven bridges. "Was it possible to take a walk through the town in such a way as to cross over every bridge once, only once (later it is considered Euler Walk)" was the question Euler had posed. Euler created the earliest known visual depiction of a modern graph after realizing

that the seven bridges and the four bodies of land were the pertinent restrictions. A modern network is made up of vertices, which are groups of points connected by edges, which are sets of connecting lines.

A graph, typically denoted  $G = (V, E)$  consists of a set of vertices  $V$  along with a set of edges  $E$ . Vertices are also referred to as points, nodes, or actors in social networks, players, or representatives. Edges are also known as curves, lines, or ties, or links in social networks. An edge  $e = (u, v)$  is defined as an unordered pair of vertices that serve as its endpoints. Two vertices  $u$  and  $v$  are considered adjacent if there is an edge  $(u, v)$  that connects them. An edge  $e = (u, v)$  that connects a vertex to itself is known as a loop. The total number of vertices in a graph is generally denoted as  $n$ , whereas the number of edges is denoted as  $m$ . The nodes are positioned in the plane, and thus the length of the lines is arbitrary unless otherwise specified.

Graph theory is a fundamental branch of mathematics, specifically within discrete mathematics, which is also known as the mathematics of networks. It has numerous applications in the field of computer science, including syntactic analysis and fault detection. Moreover, it plays a crucial role in engineering

and technology. Various tools and systems, such as medical imaging, face recognition systems, remote sensing, and optical character recognition, have been developed using graph theory. Graphs are used to model and understand complex systems with interconnected components, making graph theory an essential tool for analysing and optimizing networks, structures, and relationships.

Graph theory enables the study of fundamental concepts and properties of graphs, such as connectivity, paths, cycles, degrees, and distances. These properties help in understanding the structure and behaviour of networks and can reveal important insights into the relationships and interactions between their elements. Graph algorithms form a significant part of graph theory. These algorithms can compute various properties of graphs, such as finding the shortest path between two vertices, determining whether a graph is connected, or discovering clusters within a network. The analysis and development of efficient graph algorithms have practical applications in various domains, including transportation planning, social network analysis, routing problems, and database management.

Scientists like Albert –Laszlo Barabasi and Nicholas A. Christakis have made significant contributions to the field of network science, which includes the applications of graph theory to various systems, including biological network. Their work has influenced the exploration of network models in the context of complex biological structures, such as the human heart.

The human heart is a remarkable organ that serves as the powerhouse of the circulatory system, pumping oxygen-rich blood to all parts of the body.

Graph theory provides a powerful tool for modelling and understanding the complex network structure of the human heart. In the context of a network model of the human heart, the vertices of the graph can represent various entities, such as individual cells, cardiac tissues or anatomical regions. These vertices are then interconnected by edges, which represent the physical or functional relationships between these entities.

In recent years, graph theory has been particularly influential in image segmentation

techniques, especially in the field of medical image processing, which is currently a highly active area of research. In this paper, we present the techniques of graph theory and propose a model for a micro cardiac network system. The micro cardiac network graph represents the interconnected nodes and edges within a small-scale model of the cardiac system. It includes components such as cardiac cells, blood vessels, electrical pathways, and other relevant structures.

The primary objective is to analyse the blood flow system in the human heart, considering both oxygenated and deoxygenated blood circulation, using network graph theory. The crossing numbers, which are essential parameters for obtaining precise results in an electrical circuit represented by a planar graph, may also contribute to understanding the blood flow system in the human heart. Additionally, we introduce an algorithm for the stereographic projection of a graph to enhance the performance of the model. This researchwork will undoubtedly contribute to the development of tools for solving the blood flow system in the human heart.

### Basic Definition

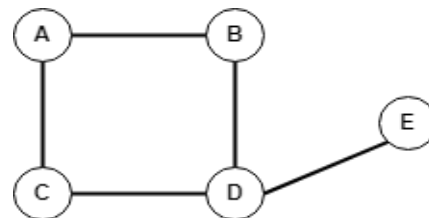
#### Graph

A graph is made up of vertices that are joined to one another by a set of edges.

An ordered pair of vertices and edges is referred to as a graph.

$$G = (V, E)$$

In this case,  $E$  is the set of edges linking the vertices, and  $V$  is the set of vertices.



Figure

$$V = \{A, B, C, D, E\}$$

$$E = \{AB, AC, BD, CD, DE\}$$

#### Sub graph

When  $v_1 \subseteq v_2$  and  $e_1 \subseteq e_2$ , a graph  $G_1 = (v_1, e_1)$  is a sub-graph of  $G_2 = (v_2, e_2)$ .

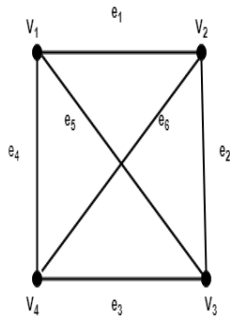


Figure: graph

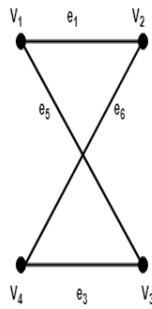


Figure: sub graph

H is present in graph G. The edges of H that contain in G and the vertex set of H that contain in G. H is therefore a sub-graph.

**Degree**

The degree of any vertex ( $v$ ) in a graph is equal to the number of edges that contain the vertex. ( $v$ ) represents the degree of  $v$ .

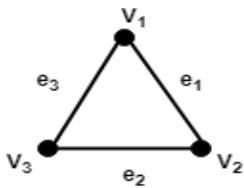


Figure: degree

Because the vertex  $v_1$  is contained by two edges,  $e_1$  and  $e_2$ , its degree is two.

**Loop**

A loop is any line that connects a point to itself. The loop is at vertex b.

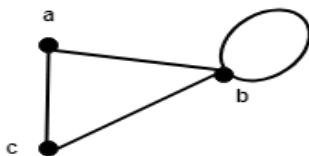


Figure: loop

**Complete Graph**

If every vertex in a basic graph with  $n$  vertices has a degree of  $n - 1$ , the graph is considered complete. In other words, a single vertex is connected to the remaining vertices in the graph or  $n - 1$  edges. Another name for a complete graph is a full graph.

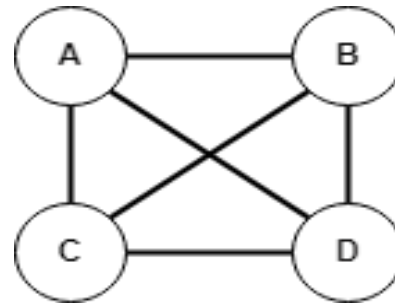


Figure: complete graph

**Multiple Edges**

When two or more edges share endpoints, they are referred to as multiple or parallel edges.

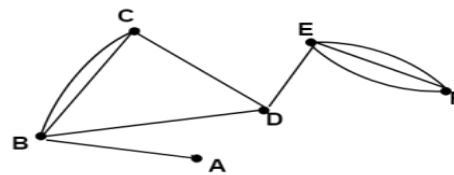


Figure Multiple Edges

**Connected Graph**

A connected graph is one in which we can go from any vertex to any other vertex. Every pair of vertices in a linked graph has at least one path connecting them.

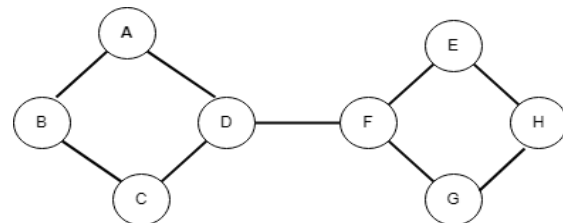


Figure Connected Graph

We are able to visit any vertex in this graph from any other vertex. Every pair of vertices has at least one path connecting them. The Graph Is Connected.

**Planar Graph**

Any graph that can be drawn in a plane when no two of its edges cross over is said to be planar.

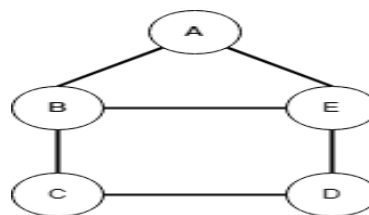


Figure Planar Graph

It is possible to draw this graph in a plane without any edges crossing. This graph is planar.

**Properties of Planar Graph**

- If a connected planar graph  $G$  has  $e$  edges and  $r$  regions, then  $r \leq \frac{2}{3}e$ .
- If a connected planar graph  $G$  has  $e$  edges,  $v$  vertices, and  $r$  regions, then  $v - e + r = 2$ .
- If a connected planar graph  $G$  has  $e$  edges and  $v$  vertices, then  $3v - e \geq 6$ .
- A complete graph  $K_n$  is a planar if and only if  $n < 5$ .
- A complete bipartite graph  $K_{mn}$  is planar if and only if  $m < 3$  or  $n > 3$ .

**Non-Planar Graph**

A non-planar graph is any graph that is not a planar graph. Stated differently, a non-planar graph is one that cannot be drawn without at least two of its crossing edges.

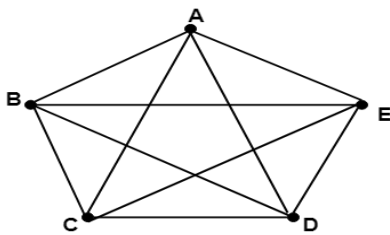


Figure: non planar

**Properties of Non-Planar Graphs**

A graph may only be classified as non-planar if it has a sub-graph that is homeomorphic to either  $K_5$  or  $K_{3,3}$ .

**Pseudo graph**

A pseudo graph is a graph that has self-loops but lacks parallel edges.

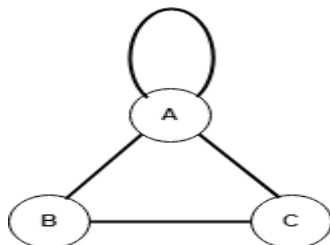


Figure: pseudo graph

There are three vertices and four edges in this graph, one of which is a self-loop. Although there are no parallel edges, there is a self-loop. It is a pseudo prime as a result.

**Bipartite Graph**

A graph with vertices that can be split into the sets  $X$  and  $Y$  is called a bipartite graph. Only the vertices of set  $Y$  and  $X$ 's vertices connect. There isn't a join between any vertex in the same set.

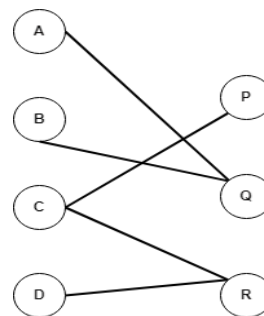


Figure: bipartite

**Complete Bipartite Graph**

A full bipartite graph is defined as ' $G$ ',  $G = (V, E)$  with partition  $V = \{V_1, V_2\}$  where each vertex in  $V_1$  is connected to each vertex in  $V_2$ . A full bipartite graph, in general, joins every vertex in set  $V_1$  to every vertex in set  $V_2$ .

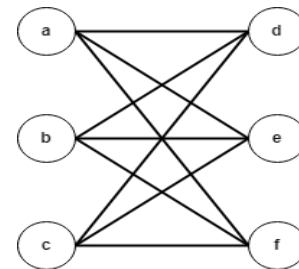


Figure: complete bipartite

The aforementioned graph is a complete bipartite graph due to the edges that link every vertex in set  $V_1$  to every vertex in set  $V_2$ .

**Hamiltonian Graph**

A connected graph is referred to as a Hamiltonian graph if there is a closed walk that visits each vertex precisely once (apart from the starting vertex) without repeating any edges.

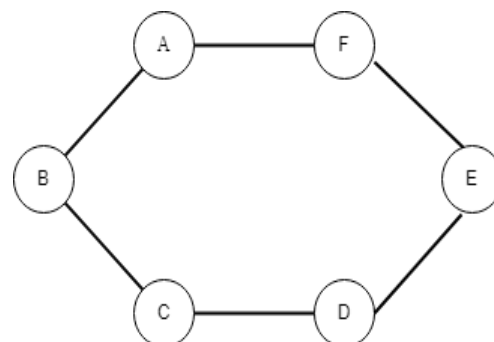


Figure: Hamiltonian

A closed walk ABCDEFG through this graph visits every vertex precisely once, with the exception of the initial vertex. This graph is Hamiltonian since all of the vertices are visited Without repeating any edges.

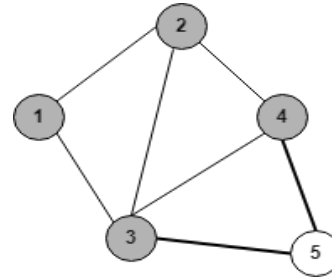


Figure: Walk 1-2-3-4-2-1-3

**Euler Graph**

An Euler graph is a linked graph with an even degree at every vertex.

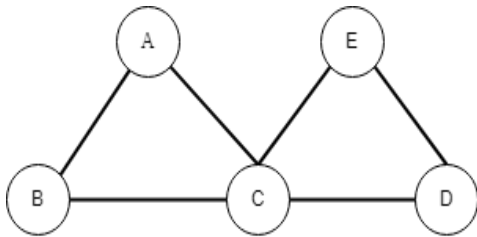


Figure: Euler graph

There are connections in this graph. Every vertex has an equal degree. It is an Euler graph as a result.

**Path**

If every vertex in a walk is unique, the walk is referred to as a path.

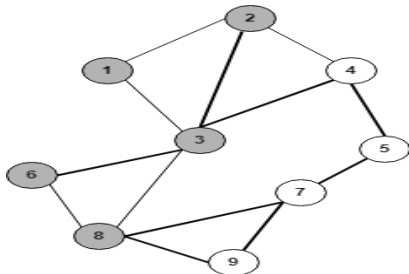
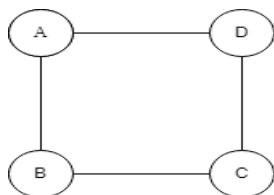


Figure: Path 6-8-3-1-2-4

**Walk**

A graph's vertex and edge succession is called a walk.



**Weights**

A weighted graph is a unique kind of graph where the edges are given weights that correspond to various relative measurement units, such as cost and distance.

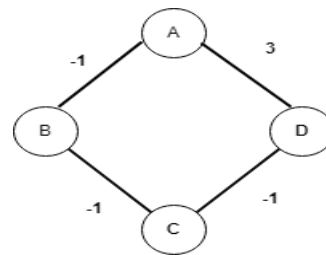


Figure : Weights

**Simple Graph**

A simple graph is defined as one that lacks both parallel edges and self-loops.

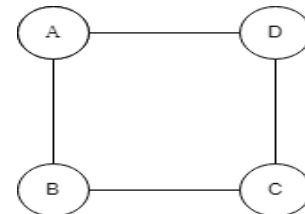


Figure: simple

There are three edges and three vertices in this graph. Neither parallel edges nor Self-loop exist. It is a basic graph as a result.

**Spanning Subgraph**

A spanning sub-graph is a sub-graph that shares the same set of vertices as the parent graph. "If  $V(H) = V(G)$ , then a subgraph  $H$  of  $G$  is a spanning sub graph of  $G$ ."

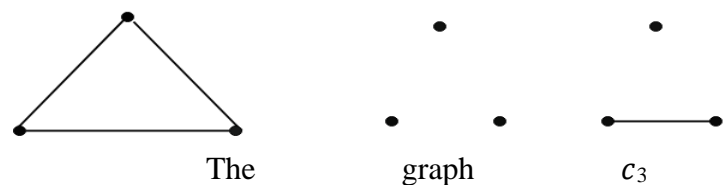


Figure : spanning sub graph of  $C_3$

**Cyclegraph**

A cycle graph is a straightforward graph

with 'n' vertices ( $n \geq 3$ ) and n edges that form a cycle of length 'n'. Every vertex in a cycle graph has a degree of two.

Every vertex in these graphs has a degree of two. They are cycle graphs as a result.

### Proposed Network Model on Human Heart

#### Crossing Numbers

The minimum number of edge crossings in a plane drawing of a graph  $G$  is its crossing number  $Cr(G)$ . For example, a graph is planar if and only if there is zero crossing in it.

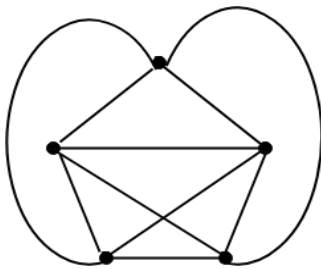


Figure: A single crossing on the graph

#### Density of A Graph

The smallest number of planar graphs into which the edges of a graph  $G$  can be divided is its thickness. That is, the thickness of  $G$  is at most  $k$  if there is a set of  $k$  planar graphs with the same set of vertices such that the union of these planar graphs is  $G$ .

#### Biplanar Graph

Bi-planar graphs are defined as the product of two planar graphs, i.e., graphs with thickness 1 or 2.

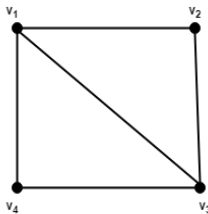


Figure:  $Cr(G) = 0 \quad t(G) = 1$

A planar graph has one thickness.

#### A Graph Projected In Stereography

A stereographic projection is a perspective representation of the sphere onto a plane perpendicular to its diameter at a particular point on the sphere.

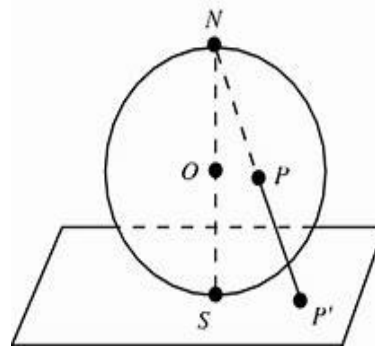


Figure: Stereography

#### Network Graph

Network graph is simply called as graph. It consists of a set of nodes connected by branches. In graphs, a node is a common point of two or more branches. Sometimes, only a single branch may connect to the node. A branch is a line segment that connects two nodes.

#### In a network graph

##### Nodes

In a graph representing the human heart, nodes can be used to represent the atria, ventricles, valves, conduction system elements within the heart.

##### Edges

Edges in the heart network graph can symbolize the connections, pathways, or relationship between different cardiac components. These connections can represent blood flow patterns. They can be directed or undirected

##### Weighted graphs

A Weighted graph can assign numerical values to the edges to represent factors such as blood volume, electrical conduction velocities, or the strength of connections between different components of the heart.

##### Multi-graph

A Multi-graph representing the heart can account for multiple types of interactions or pathways within the cardiac system, such as electrical conduction pathways and blood flow pathways.

#### Human Heart

One of the most vital organs needed to keep life alive is the human heart. It has four chambers and is a muscular organ. The heart resembles a tightly balled fist in size. The human

heart is situated in the thoracic cavity, somewhat to the left of the sternum, between the lungs. Any organism's heart's job is to keep the blood flowing through the body continuously. As a result, the cells and tissues receive fresh oxygen and nutrients.

The human heart's main job is to circulate blood throughout the body. The heart also keeps the body's blood pressure at a healthy level. The part of circulation called pulmonary circulation is in charge of returning oxygenated blood to the heart after transporting deoxygenated blood has left the heart and entered the lungs. Another type of circulation is called systemic circulation, in which blood that has been oxygenated is pumped from the heart to all of the body's organs and tissues and then returns to the heart deoxygenated. The fibrous membrane that surrounds the heart externally is called the pericardium.

Three layers comprise the heart wall: the epicardium, myocardium, and endocardium.

Numerous blood arteries network outward from the heart's exterior structure, while other significant vessels emerge from within. Usually, the blood vessels consist of the following: Capillaries, arteries, and veins. The flaps of fibrous tissue that are seen in the heart chambers in between the veins are called valves. They guarantee that blood moves only in one direction. The aorta, vena cava, and pulmonary arteries are attached to the base of the heart at its superior end. The bottom end of the heart is called the apex, and it is situated just above the diaphragm. The peak of the heart is oriented toward the left, and the lower portion of the heart is situated along the body's midline. Since the heart is located mostly on the left side of the body, about two thirds of its mass is on the left side and the remaining third is on the right. The body receives about 6,000–7,500 liters of blood every day from the heart. The heart beats 100,000 times a day on average.

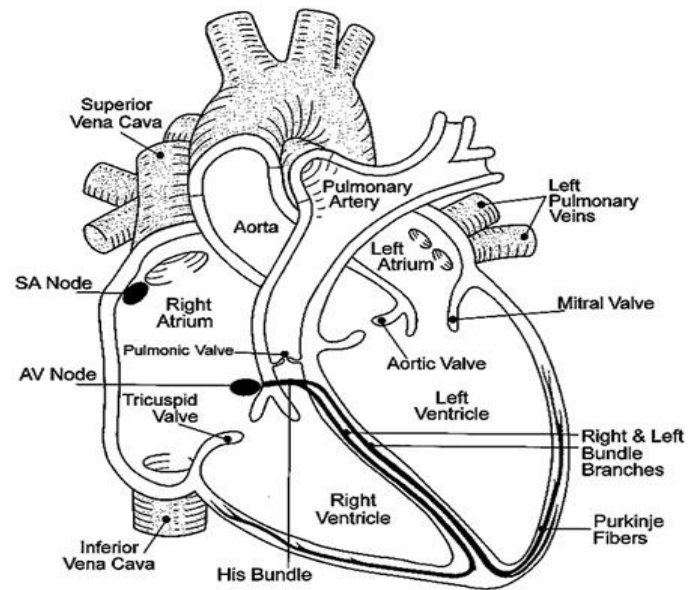


Figure: Human heart

### Proposed Model

For the human heart, we suggest a network model based on graph theory. We have attempted to use graph theory to visualize the human heart as best we can.

#### Crossing Numbers

The minimal number of crossings that any edge of graph  $G$  has among all feasible drawings of  $G$  in the plane is its crossing number, or  $n(G)$ . These crossing numbers are essential for getting accurate results in planar graph-represented electrical circuits. The crossing numbers of complete graphs and full bipartite graphs have been studied in great detail. In this work, the stereographic projection graph and a graph's thickness are observed using crossing numbers.

The crossing number can be used to analyse the complexity of the connections between different parts of the heart's network.

In the context of the human heart, the graph could represent the network of blood vessels, electrical pathways, or other interconnected components. By determining the crossing number, we can understand the level of complexity in terms of the number of interconnections or interactions between different parts of the heart.

The crossing number can help in visualizing and optimizing the layout of the graph representation. Minimizing the crossing numbers can lead to clearer and more

understandable visualizations of the relationships between different components of the heart, such as nodes representing different cardiac structures or functions.

The graphs in figures have one crossing, two crossings, and three crossings.

**Algorithm**  $n(K_5) = 1$

*Step 1:* Start by looking at a graph in the plane that has  $k_5$  vertices and  $C = n(K_5)$  crossings.

*Step 2:* Examine the case shown in figure 3.6.1(a), where two edges cross at most once and then at the same location.

*Step 3:* It follows that  $c$  is greater than or equal to 1 since  $k_5$  is co-planar.

*Step 4:* To build a linked plane graph  $G$  with  $5 + C$  vertices and  $10 + 2C$  edges, add a new vertex at each crossing.

Then, we know that

$3v - e \geq 6$  if a linked planar graph  $G$  contains  $e$  edges and  $v$  vertices. Afterward, we have

$$\begin{aligned} 3v - e &\geq 6 \\ e &\leq 3v - 6 \\ 10 + 2C &\leq 3(5 + C) - 6 \\ 10 + 2C &\leq 15 + 3C - 6 \\ 10 + 2C &\leq 9 + 3C \\ 10 - 9 &\leq 3C - 2C \\ 1 &\leq C \\ C &\geq 1 \end{aligned}$$

$C = n(K_5) > 1$

*Step 5:* Observation reveals that there is a drawing of  $k_5$  with a single one crossing, which supports the idea that  $n(K_5) = 1$ ,

Where  $k_5$  is a complete graph with 5 vertices and  $C$  represents the number of crossings.

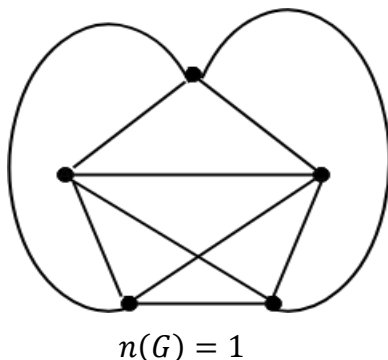


Figure: One crossover in the graph

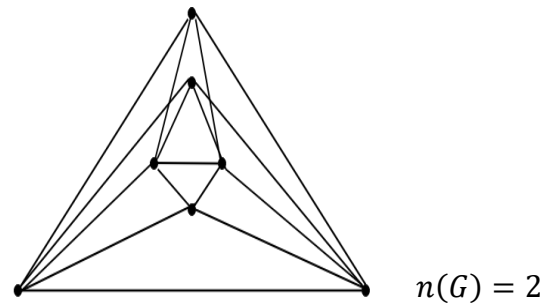
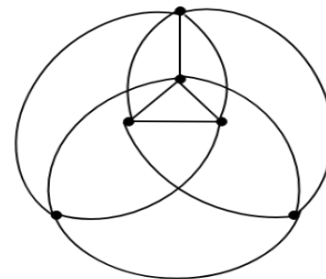


Figure: A two-crossing graph



$n(G) = 3$

Figure: A three-crossing graph

**Density of A Graph**

The least number of planar sub-graphs whose union is the specified graph  $G$  is the definition of a graph's thickness. In 2002, NARSINGH DEO presented this idea. A graph's thickness is denoted by the symbol  $t(G)$ .

A non-planar graph  $G$  fulfill  $t(G) \geq 2$ , whereas a planar graph  $G$  has a thickness of one, or  $t(G) = 1$ . Bi-planar graphs are thickened graphs with a thickness of two.

The concept of thickness can be used to analyse the physical space required for the connections between different parts of the heart's network. Thickness of edges in a graph can represent the strength or weight of the connections between nodes such as blood vessels, chambers, or valves in a network. In human heart network model, thickness of edges could potentially represent the strength of the electrical or mechanical connections between different parts of the heart, which could be important for understanding the dynamics of the heart's function.

The concept of thickness in graph theory is applied to graphs with a planar embedding,

meaning they can be drawn without any edge crossings in two-dimensional space.

**Specific Graphs**

A graph's bottom constraint on thickness is provided by  $t(G) \geq \lceil m/(3n - 6) \rceil$  where  $n \geq 3$  denotes the number of vertices and  $m$  denotes the number of edges. The entire graph's thickness on  $n$  vertices,  $k_n$  equals

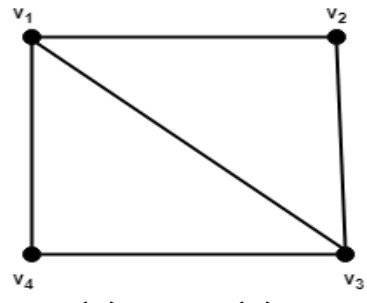
$\lceil \frac{n+7}{6} \rceil$ , With the exception of  $n=9, 10$ , in which case the thickness is 3.

the thickness of a complete bipartite graph  $K_{a,b}$  is typically  $\lceil \frac{ab}{2(a+b-2)} \rceil$ , with a few outliers.

The thickness of  $K_{n,n}$  is given by  $t(K_{n,n}) = \lceil \frac{n+5}{4} \rceil$

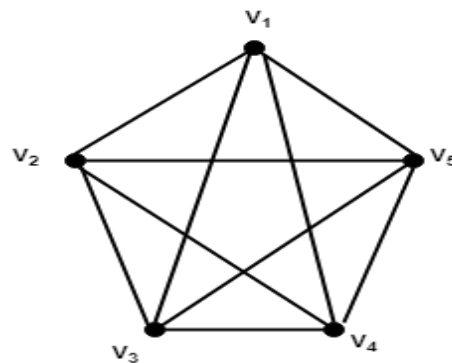
For instance, a full graph with eight vertices has a thickness of two. In contrast, a full graph with nine vertices has three layers.

It is typically challenging to determine the thickness of an arbitrary graph, despite the availability of multiple experimental data. In figure 3.6.5(c), we have ascertained the thickness of the entire graph  $k_8$ .

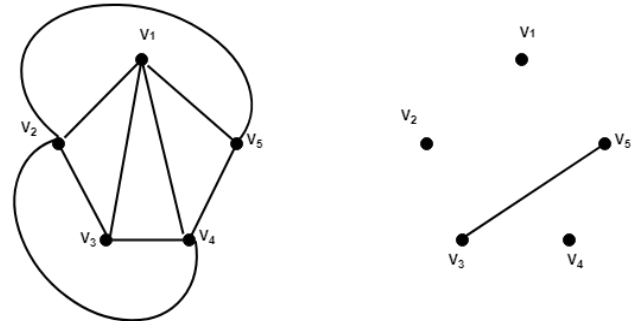


$Cr(G) = 0 \quad t(G) = 1$

A planar graph has one thickness.



**K5**



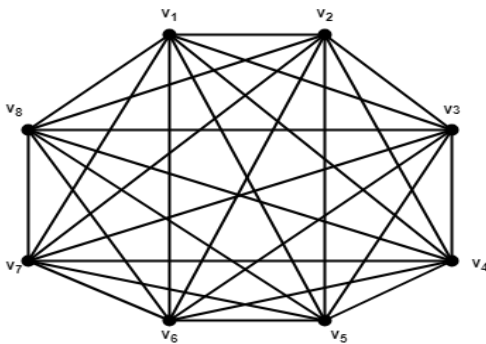
$t(G) = 2$

The entire graph  $k_5$  has a thickness of 2.

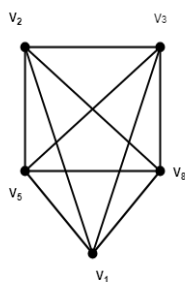
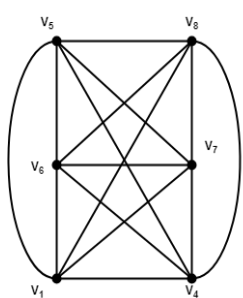
**A Graph Projected In Stereography**

A mapping of points from a sphere to a plane is called a stereographic projection. The graph can be represented in three dimensions using a stereographic projection. A graph placed on the plane has the same properties as one placed on the sphere.

Stereographic projection is commonly used in network models of the human heart because it allows for a three-dimensional representation of the complex network of interconnected cardiac cells and tissues in a two-dimensional space. This visualization technique helps researchers and medical professionals



**K8**



First sub graph of  $k_8$  Second sub graph of  $k_8$

better understand the structural and functional relationships within the heart, and can aid in the analysis of electrical conduction pathways, propagation of cardiac impulses, and the development of arrhythmias.

Euler's formula, coined by Leonhard Euler in the XVIII th century, is one of the most famous and beautiful formulas in the mathematical world. Euler's formula is commonly used in graph theory to analyse and understand the connectivity and relationships within a network or graph. the stereographic projection of a graph in a network model of the human heart, Euler's formula can help us understand the topology and structure of the network, as well as the relationships between different parts of the heart.

In a micro cardiac network graph, vertices could represent individual cells or nodes in the network, edges could represent connections between cells or nodes, and faces could represent enclosed regions within the network. By applying Euler's formula, you can analyse the connectivity and topology of the micro cardiac network graph

This can help us better understand the functioning and dynamics of the heart. using Euler's formula in stereographic projection of a graph in a network model of the human heart can provide valuable information and understand the heart structure and function. Euler formulated the theorem for simply connected graphs in 1752, hence the term Eulerian graph or a connected planar graph

**Algorithm: Euler's Formula**  $n - m + f = 2$

*Step 1:* We'll move forward with the induction approach.

*Step 2:* Next,  $n = 1, f = 1$ , and  $n - m = 2$  if  $m = 0$ . As a result, when  $m = 0$ , the outcome is valid.

*Step 3:* Let us now assume that the result holds for all linked graphs with  $k - 1$  edges, where  $k$  is greater than or equal to 1.

*Step 4:* Assume that  $G$  contains  $k$  edges,  $f$  regions, and  $n$  vertices. To demonstrate that  $n = k + 2$

*Step 5:* Given a tree  $G$ ,  $n = k + 1$ . Given that  $f = 1$ , we have

$n - k + f = (k + 1) - k + 1 = 2$ , it provides the intended outcome.

*Step 6:* There must be an edge  $e$  in  $G$  that is a part of a cycle if  $G$  is not a tree. Since  $G - e$  has  $f - 1$  regions and order  $n$ , it is a linked plane graph of size  $k - 1$ .

*Step 7:* After changing the parameters,  $n - (k - 1) + (f - 1) = 2$  is the result. This validates the outcome.

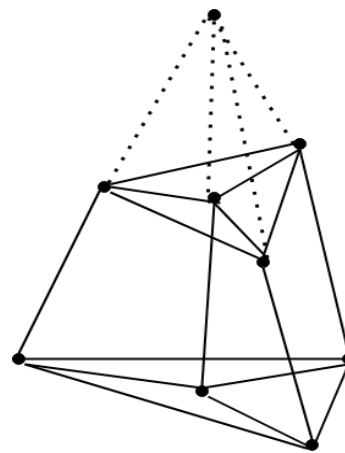


Figure: Stereographic projection of a graph

**Network System for Human Microcardiacs**

The heart is a muscular organ that collects blood from all parts of the body that has lost oxygen and pumps it to the lungs, where oxygen is added and carbon dioxide is exhaled. It then transports blood that has been oxygenated by the lungs to every part of the body.

The heart pumps around 7,200 liters of blood throughout the body in a 24-hour period. The heart is located in the center of the chest and has a slight leftward tilt. The heart beats three billion times in a lifetime, or about 100,000 times a day on average. An adult's heart beats between 60 and 80 beats per minute, whereas a newborn baby's heart beats between 70 and 190 beats per minute.

**In Right Side Heart Work**

Dirty blood is deposited in the right atrium of the heart via the superior and inferior vena cava.

The tricuspid valve allows blood to pass from the contracting right atrium into the right ventricle.

As the right ventricle fills with blood, the tricuspid valve closes, preventing blood from returning to the right atrium.

Blood is pushed through the pulmonary valve and into the pulmonary artery, which carries it to the lungs, as the right ventricle contracts.

In the lungs, the blood is purified, filled with oxygen, and free of carbon dioxide.

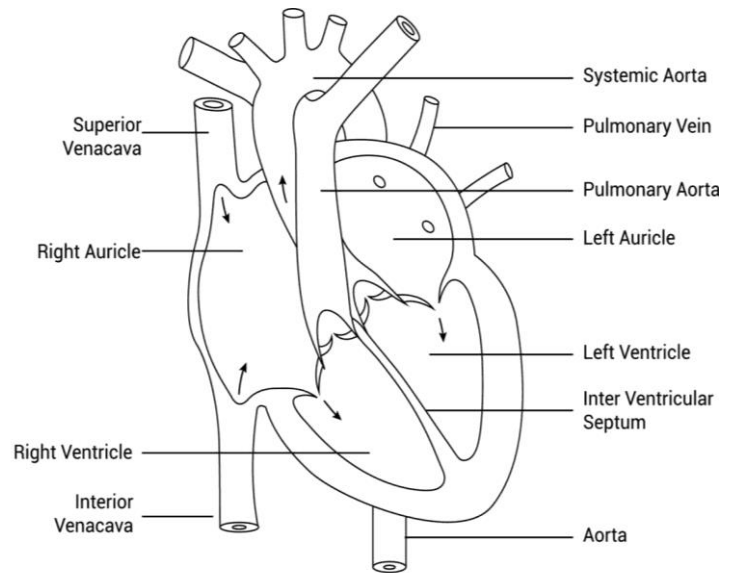
**In Left Side Heart Work**

Through the pulmonary vein, the blood that carries oxygen is returned to the left atrium.

The mitral valve allows blood to enter the left ventricle when the left atrium contracts.

When the left ventricle fills with blood, the mitral valve completely shuts, stopping blood flow.

The aortic valve opens and oxygen-rich blood exits the heart through the aorta when the left ventricle contracts.



**Human Heart**

Figure: Microcardiac network system in humans

**Graph for Micro Cardiac Network**

The Micro cardiac network graph is a visual representation of the complex network of interactions between different components (nodes) in the cardiac system at a microscopic level.

The micro cardiac network graph represents the interconnected nodes and edges within a small-scale model of the cardiac system. It includes components such as cardiac cells, blood vessels, electrical pathways, and other relevant structures.

**Nodes**

*Cardiac Cells*

Representing the fundamental units responsible for generating and transmitting electrical signals within the heart. The nodes represents the nucleus and edges represents the intercalated disk, branching cell.

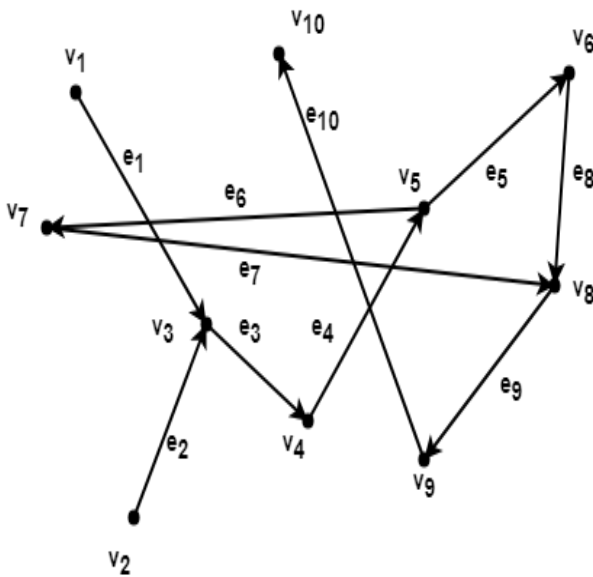


Figure:  $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$   
 where  $v_1$ = superior vena cava,  $v_2$ = inferior vena cava,  $v_3$ =tricuspid valve,  $v_4$ =pulmonary valve,  $v_5$ =the branching point of the two pulmonary arteries,  $v_6$ = pulmonary artery,  $v_7$ =pulmonary artery,  $v_8$ =mitral valve,  $v_9$ =aortic valve,  $v_{10}$ = aorta and

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

The set of directed edges which represented the path of blood in the heart.

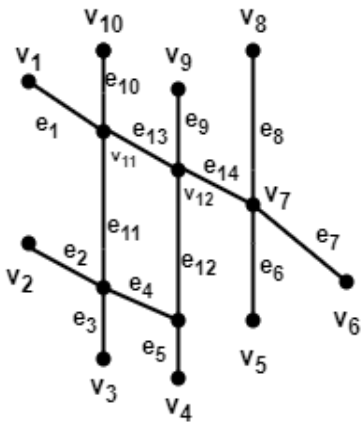


Figure: Cardiac cells

**Blood Vessels**

Depicting the network of arteries, veins, and capillaries that supply oxygenated blood to the cardiac cells.  $v_1$  and  $v_2$  represents the artery,  $v_3$  and  $v_4$  represents the vein and  $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$  represents the capillary.

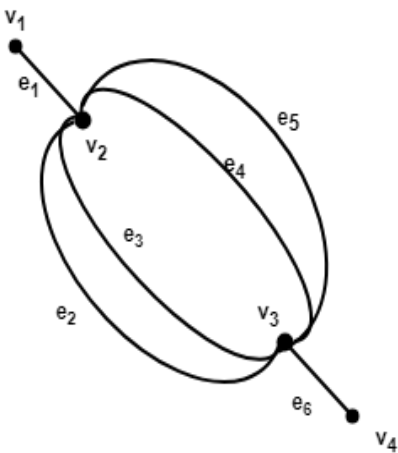


Figure: Blood vessels

**Electrical Pathways**

Showing the routes through which electrical impulses travel to coordinate the contractions of heart muscles.

$v_1$ = Sino-atrial node,  $v_2$ = Atrio-ventricular node,  $c_1$ = bundle of His (left bundle) and  $c_2$ = bundle of His (right bundle)

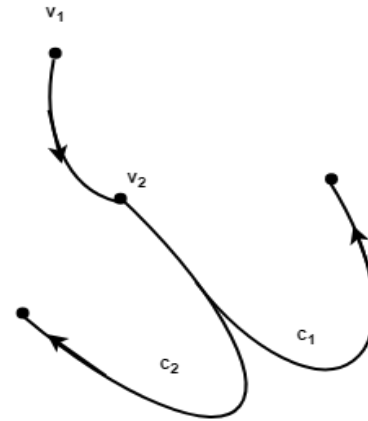


Figure: Electrical pathways

**Edges**

*Connections*

The edges in the micro cardiac network graph illustrate the connections between different nodes.

*Signal Transmission*

They symbolize the pathways through which signals, nutrients, and other substances are exchanged between cardiac components.

$v_1$ = Sino-atrial node,  $v_2$ = Atrio-ventricular node,  $e_1$ = Bachmann's bundle,  $e_2$ = middle intermodal tract,  $e_3$ = posterior intermodal tract,  $e_4$ = anterior intermodal tract,  $e_5$ = conduction pathways,  $v_3, v_4, v_5, v_6$ = perkinje fibres.

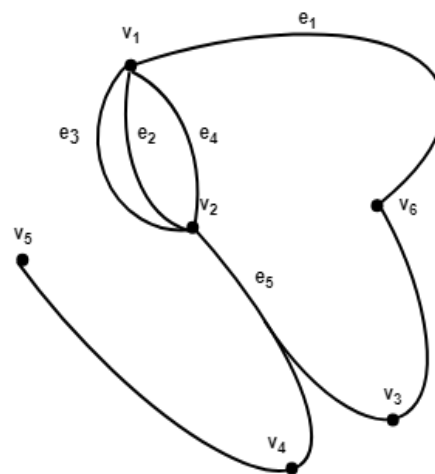
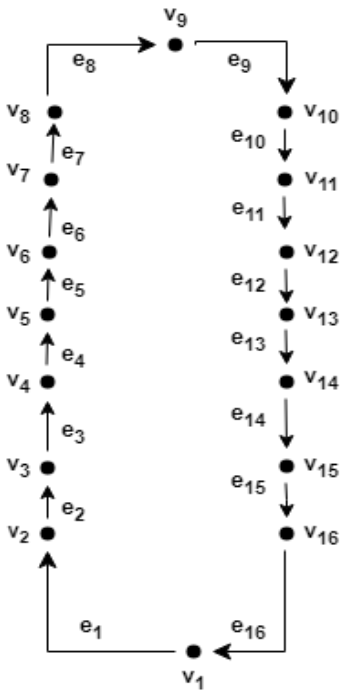


Figure: Signal transmission

Natural or predetermined blood circulation occurs in the human heart system. As a result, we have investigated and evaluated the cardiac network system's planarity as well as its Hamiltonian or Eulerian graphs.



Deoxygenated blood      Oxygenated blood

$V$   
 $= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$

Where  $v_1$ =systematic capillaries( in the body),  $v_2$ = veins,  $v_3$ =vena cava,  $v_4$ = right atrium,  $v_5$ =tricuspid valve,  $v_6$ =right ventricle,  $v_7$ = semi-lunar valve,  $v_8$ = pulmonary artery,  $v_9$ =pulmonary capillaries(in the lungs),  $v_{10}$ =pulmonary vein,  $v_{11}$ =left atrium,  $v_{12}$ =bicuspid valve,  $v_{13}$ =left ventricle,  $v_{14}$ =semi-lunar valve,  $v_{15}$ =Aorta,  $v_{16}$ =Arteries.

$E$   
 $= \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$

Where  $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$  represents the deoxygenated blood and

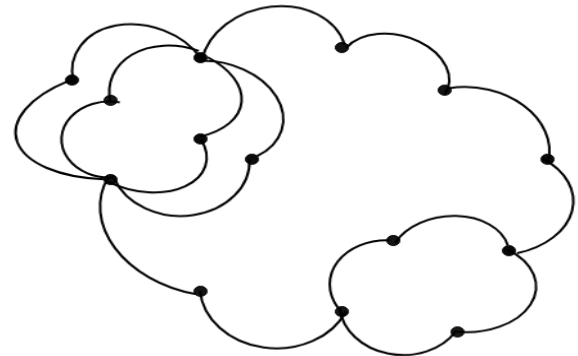
$E = \{e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$  represents the oxygenated blood

We have altered the heart's blood circulation system in Figure 3.6.5(b) to demonstrate a number of uses. We have collected connectivity graphs of the human body's infrastructure, concentrating on the heart's blood circulation system; these graphs we have named "Micro-cardiac Network graphs." We have also investigated this idea in connection with network graphs.

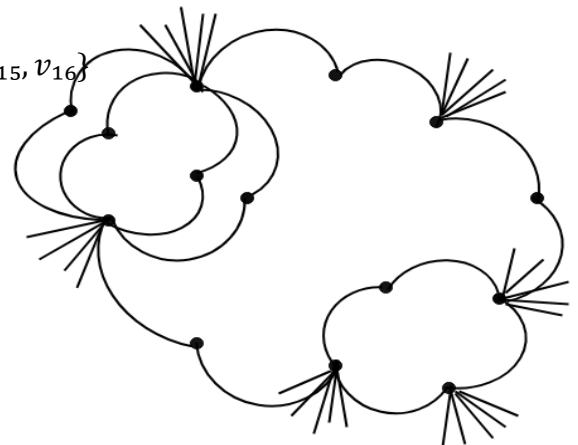
We eventually came to the decision that we should present some sketches of this system

in order to prove that a Hamiltonian or Eulerian route system exists in the functioning of the human heart.

We have identified potential remedies to the problems caused by the densely coupled micro cardiac network system. The cardiac network system may benefit from the increased edge connection.



Graph for micro cardiac network



A densely packed microcardiac network

### Conclusion

Recent developments and real-world applications in the microcardiac system have been illustrated by the graph theory, which also showcases the methods used and the outcomes. The proposed application highlights the importance of modelling the cardiac system by employing network graph theory to provide efficient solutions for stereographic network models and the human cardiac system. This study's main goal is to investigate the effects of using network graph theory. Several worthwhile results have been obtained by applying this approach. Cross numbering and stereographic projection were used in the inquiry, and the results were vital for finding solutions in electric

circuits. The outcomes of using graph techniques on the cardiac system have shown to be quite helpful in addressing problems with the human heart's blood flow system. Graph theory provides us with a tool to deal with both generality and imprecision, which helps us in our research of the heart blood flow system.

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