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Research Article

ASingle Server Queue with Limited Capacity And A Flexible Service Rate

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Abstract

The arrival process of the model under studies in this paper is Poisson with state-dependent parameters, and the service time distribution is negative exponential with state-dependent rate. If there are n customers in the system, then the arrival rate isλ_n , and the service rate is μ_n. When a customer finds a server free, they enter the service station immediately; if not, they enter a queue of size N (N>0). In the waiting line, the customer follows to the First Come, First Served policy. During the customer's service period, if they are satisfied with the service at any point during the service period, they will either continue with probability Δ or leave the system with probability 1 - Δ. At this point, the service rate becomes$\frac{1}{\Delta}$ (0<Δ $\leq$ 1).The group transformed inverse of the infinitesimal generator has been used for the analysis of this model, which is defined using the infinitesimal generator matrix. We derive the steady-state probabilities analytically using the group generalized inverses. On the basis of the same, performance metrics are generated. Moreover, a few calculations are given.

Introduction

Queues are the part of everyday life. We all wait in queues to buy a movie ticket, to make bank deposit, pay for groceries, mail a package, obtain a food in a cafeteria, to have ride in an amusement park and have become adjustment to wait but still get annoyed by unusually long waits. The queuing models are very helpful for determining how to operate a queuing system intheeffectivewayiftoomuchservicecapacitytoope ratethesysteminvolvesexcessivecosts.The models enable finding an appropriate balance between the cost of service and the amount of waiting. Queues can occur whenever resources are limited. Some queuing is tolerable in anybusinesssinceatotalabsenceofaqueuewouldsu ggestacostlycapacity.Queuingtheoryaimsto design balanced systems that serve customers quickly and efficiently but do not cost too much to be sustainable. At its most basic level, queuing theory involves an analysis of arrivals at a facility, such as a bank or a fast-food restaurant, and an analysis of the processes currently in place to serve them. The end result is a set of conclusions that aim to identify any flaws in the system and suggest how they can be ameliorated.

The origin of queuing theory can be traced to the early 1900s in a study of the Copenhagen telephone exchange by Agner krarup Erlang, A Danish engineer, statistician, and a mathematician. His work led to the Erlang theory of efficient networks and the field of telephone network analysis.

In this paper, we have made the provision of flexible servers in the sense that according to the work load in the system the servers join the service facility to serve the customers and all the servers that work in the system have heterogeneous service rates. In one hand, our investigationdiffersfromtheworkdoneonthefieldb yseveralauthorsisthatwehavetimevaryingheterog enous arrival and service rates, on the other hand, time-dependent analysis of finite capacity queuing model has been made with the help of numerical approximation technique.

In an M\M\1 queuing system, the arrival and service rates are constants. But in may practical. Situations, the arrival and service rates are state-dependent. That is, the assumption of independence and constant arrival and service rates can be relaxed to the extent of making each of the rates as $\boldsymbol{\lambda}_n$and $\boldsymbol{\mu}_n$respectively, where n is the number of customers in the system. Some exceptional cases that received considerable attention based on physical scenarios in the literature are:

$$\boldsymbol{\lambda_n} = \begin{cases} \boldsymbol{n\lambda} , & \boldsymbol{n \geq 0} \\ \frac{\lambda}{n+1}, & \boldsymbol{n \geq 0} \end{cases}$$

$$\boldsymbol{\mu_n} = \begin{cases} \boldsymbol{n\mu} , & \boldsymbol{n \geq 0} \\ \boldsymbol{\mu} , & \boldsymbol{n \geq 0} \end{cases}$$

Definitions

Arrival

Arrival defines the way customer senter the system. Mostly the arrivals are random with random intervals between the two adjacent arrivals. The arrival is described by a random distribution of intervals also called arrival pattern. The statistical pattern of the arrival can be indicated through the probability distribution of the number of the arrivals in an interval.

Server

It is a mechanism through which service is offered.

Waiting Time in the System (W_s)

It is the total time spent by a customer in the system. It can be calculated as follows:

Waiting time in the system =Waiting time In queue +Service time

Queue Length (L_q)

Number of persons waiting in the system at any time is defined as queue length.

Single Server Model

A single server serves customers one at a time from the front of the queue, according to a first-come, first-served discipline. When the service is complete the customer leaves the queue and the number of customers in the system reduces by one.

Methodology

Let (X_n, Y_n), n≥ 0 be a Markov process on the state S = $\{(n, j): 0 \leq n \leq N, 1 \leq j \leq a_n\}$with the following bock tridigonal infinitesimal generator.

$$I = \begin{bmatrix} B_0 & A_0 & 0 & 0 & \dots & \dots & 0 & 0 \\ C_1 & B_1 & A_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & C_2 & B_2 & A_2 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & C_{N-1} & B_{N-1} & A_{N-1} \\ 0 & 0 & 0 & \dots & \dots & 0 & C_N & B_N \end{bmatrix}$$

Where B_0, B_1,…….$B_N$are square matrices of order $a_0, a_1 \dots\dots, a_N$ respectively. Their diagonal elements are strictly negative, the other elements are non-negative. The matrices $A_0, A_1, A_2 , \dots\dots\dots A_{N-1}, C_1, C_2, \dots\dots\dots C_N$ are rectangular matrices and non-negative. The row sums of I are equal to 0. That is,

$B_0\, t + A_0 t = 0$

$C_i t + B_i t = 0: 1 \leq$ i $\leq$ N-1,

C_Nt + B_Nt = 0

Where t denotes the column vector and unit elements.

For the determination of the stationary probability distribution, the following realization of the Markov chain is useful. Observe the process Q during the interval of time spent at the level n, before the original process enters the level n+1 for the first time. Denote h_n , be the realization of the process. the state space of h_n is S_n = $\{(n, j): 1 \leq j \leq a_n\}$. All h_n, 0≤ n ≤ N-1 are transient Markov chains. The process h_N is the realization of process I with state space S_N= $\{(N, j): 1 \leq j \leq a_N\}$,it is an ergodic Markov chain. Denote I_n as the infinitesimal generator of the process h_n, 0≤ n ≤ N. Let M = $(m_0, m_1, m_2, \dots . m_N)$be the probability vector, where M_n be the probability that there are n customer in the system.

Algorithm

Based on the method described in the above section the following algorithm is used to find the analytic solution for the model defined in the section 2.

Step: 1 Write W=$\begin{pmatrix} G, & o \\ e' & \propto \end{pmatrix}$

Where W= -I, G is $(n-1) \times (n-1)$ matrix.

Step: 2

Check the rank of (I) = rank of (I^2), if it is true I^* exists.

Step: 3

Calculate $q' = e'G^{-1}$ and $\beta = 1 - q'j$ where β is non − zero.

Step: 4

Calculate the probability distribution $M' = \frac{1}{\beta}[-q', 1]$.

Model

The model has the following characteristics:

The queueing model has a single server with a finite waiting line of size N. The arrival process is Poisson process with state-dependent parameters. Service time distribution is negatively exponential with state-dependent rate. If there are n customers in the system then, the arrival rate $\lambda_n$and service rate μ_n.When an arrival finds the service free, the customer immediately enters the service station, otherwise, the customer enters into a queue of size N (N>0). In the waiting line, the waiting customer applies the First come First service discipline.

During the service period of the customer, if the customer is satisfied with the service at any instant of the service period, the customer leaves the system with probability Δ or continues with probability 1-Δ, the service rate becomes $\frac{1}{\Delta}$(service rate) ($0 \le \Delta \le 1$)

The Markov chain related to the model defined above is $\{(X_n, Y_n): n \ge 0\}$ with state space

$S = \{(i,j): 0 \le i \le N, j = a_0, a_1, a_2\}$

The I matrix is,

$$I = \begin{bmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \dots & \dots & 0 & 0 \\ \frac{\mu_1}{\Delta} & -\left(\lambda_1 + \frac{\mu_1}{\Delta}\right) & \lambda_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{\mu_2}{\Delta} & -\left(\lambda_2 + \frac{\mu_2}{\Delta}\right) & \lambda_2 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \frac{\mu_{N-1}}{\Delta} & -(\lambda_{N-1} + \frac{\mu_{N-1}}{\Delta}) & \lambda_{N-1} \\ 0 & 0 & 0 & \dots & \dots & 0 & \frac{\mu_N}{\Delta} & -\frac{\mu_N}{\Delta} \end{bmatrix}$$

Where $a_0 = \{0\}$, $a_1 = \{1,2,3,\dots N-1\}$, $a_2 = \{N\}$.

Define $W = \begin{pmatrix} G & o \\ e' & \alpha \end{pmatrix}$,

Where W = - I, G is an (n-1)×(n-1) matrix, corresponding to the states $\{0,1,2,\dots.N\}$.

The Mathematical Model And Analysis

The model defined in the article can be identified using the above construction. For the solution, we use the technique of group inverse.

The Markov chain related to the model discussed in this article is $\{(X_n, Y_n): n \ge 0\}$ with state space $S = \{(i,j): 0 \le i \le N, j = a_0, a_1, a_2\}$.

To apply the algorithm of section 3, we take $W = \begin{pmatrix} G & o \\ e' & \alpha \end{pmatrix}$, where W = - I

The I matrix is I =

$$\begin{bmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \dots & \dots & 0 & 0 \\ \frac{\mu_1}{\Delta} & -\left(\lambda_1 + \frac{\mu_1}{\Delta}\right) & \lambda_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{\mu_2}{\Delta} & -\left(\lambda_2 + \frac{\mu_2}{\Delta}\right) & \lambda_2 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \frac{\mu_{N-1}}{\Delta} & -(\lambda_{N-1} + \frac{\mu_{N-1}}{\Delta}) & \lambda_{N-1} \\ 0 & 0 & 0 & \dots & \dots & 0 & \frac{\mu_N}{\Delta} & -\frac{\mu_N}{\Delta} \end{bmatrix}$$

Where $a_0 = \{0\}$, $a_1 = \{1,2,3,\dots N-1\}$, $a_2 = \{N\}$.

G =

$$\begin{bmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \dots & \dots & 0 \\ \frac{\mu_1}{\Delta} & -\left(\lambda_1 + \frac{\mu_1}{\Delta}\right) & \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \frac{\mu_2}{\Delta} & -\left(\lambda_2 + \frac{\mu_2}{\Delta}\right) & \lambda_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \frac{\mu_{N-1}}{\Delta} & -(\lambda_{N-1} + \frac{\mu_{N-1}}{\Delta}) \end{bmatrix}$$

$$O = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \dots \\ \dots \\ -\lambda_{N-1} \end{pmatrix}$$

$e' = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & -\frac{\mu_N}{\Delta} \end{pmatrix}$

$\alpha = (\mu_N)$

We analyse the above model using the method described in section 4.

G^{-1} is given by

$G^{-1} = (g_{ij})_{(N-1)\times(N-1)}$

Where,

$g_{ij} = \frac{1}{\lambda_{j-1}}\left[1 + \sum_{k=1}^{N-2}\prod_{i=j}^{k}\frac{\mu_i}{\Delta\lambda_i}\right]$, for i≤ j.

$g_{ij} = \frac{1}{(\lambda_j - 1)}\sum_{k=i-j}^{N-2}\prod_{i=j}^{k}\frac{\mu_i}{\Delta\lambda_i}$, for i > j.

The unique fixed probability vector $M = \frac{1}{\beta}(M_0, M_1, M_2, \dots\dots, M_N)$

$$\beta = \frac{1}{\prod_{i=0}^{N-1}\lambda_i}\left[\prod_{i=1}^{N}\frac{\mu_i}{\Delta} + \sum_{k=1}^{N-1}\left(\prod_{i=0}^{k-1}\lambda_i\prod_{j=k+1}^{N}\frac{\mu_j}{\Delta}\right) + \prod_{i=0}^{N-1}\lambda_i\right]$$

Where,

$M_0 = \frac{1}{\Delta k}\prod_{j=1}^{N}\mu_j$

$M_1 = \frac{1}{k}\prod_{j=0}^{i-1}\lambda_j \prod_{j=i+1}^{N}\frac{\mu_j}{\Delta}$

$M_N = \frac{1}{k}\prod_{j=0}^{N-1}\lambda_j$

Where,

$$K = \left[\prod_{i=1}^{N}\frac{\mu_i}{\Delta} + \sum_{k=1}^{N-1}\left(\prod_{i=0}^{k-1}\lambda_i \prod_{j=k+1}^{N}\frac{\mu_j}{\Delta}\right) + \prod_{i=0}^{N-1}\lambda_i\right]$$

Some Performance Measures

In this section, we present the following performance measures related to the model discussed in this paper.

The mean number of customers in the system: L = $\sum_{n=1}^{N} nM_n$

The probability that the server is idle: M_0

The blocking probability: h_β = 1 - M_N

Numerical Analysis

In this segment, we examine three specifics (denoted by mode I, model II and model III) related to the queueing system discussed in this article. In the study, we take $\lambda_n = \frac{1}{n+1}$, $\mu_n = \frac{1}{2n}$ and vary the values of N=5(Model I), N=7(Model II), N=10(Model III).

Model I:

$\mathbf{M^{(n)}\backslash M^{(n)}\backslash 1/5}$

Δ = 0.9

Arrival Rates (μ)	0.92	0.81	0.71	0.61	0.53	0.44	0.38
Service Rates (λ)	1	0.86	0.73	0.63	0.55	0.50	0.46

W = - I =

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 & 0 \\ 0 & 0 & 0 & -0.678 & 1.228 & -0.55 \\ 0 & 0 & 0 & 0 & -0.589 & 0.589 \end{pmatrix}$$

$$G = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 \\ 0 & 0 & 0 & -0.678 & 1.228 \end{pmatrix}$$

is (n-1)×(n-1) matrix

$$G^{-1} = \begin{pmatrix} 7.60 & 6.60 & 5.20 & 3.54 & 1.81 \\ 6.60 & 6.60 & 5.20 & 3.54 & 1.81 \\ 5.44 & 5.44 & 5.20 & 3.54 & 1.81 \\ 4.00 & 4.00 & 3.83 & 3.54 & 1.81 \\ 2.21 & 2.21 & 2.11 & 1.95 & 1.81 \end{pmatrix}$$

Rank of I = Rank of I^2 = 5

e' = (0 0 0 0 -0.589)

q' = (-1.3017 -1.3017 -1.2428 -1.1486 -1.0661)

β = 1- q'j

=5.912

$M' = \frac{1}{\beta}[-q', 1]$.

= (0.220 0.220 0.210 0.194 0.180 0.169)

The mean number of customers in the system is,

L = $\sum_{n=1}^{N} nM_n$

= $M_1 + 2M_2 + 3M_3 + 4M_4 + 5M_5$

= 2.787

The probability that the server is idle,

M_0 = 0.220

The blocking probability,

h_β = 1 - M_N

= 0.831

Model II:

$\mathbf{M^{(n)}\backslash M^{(n)}\backslash 1/7}$

Δ = 0.9

Arrival rates (μ)	0.92	0.81	0.71	0.61	0.53
Service rates (λ)	1	0.86	0.73	0.63	0.55

W = - I =

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.678 & 1.228 & -0.55 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.589 & 1.089 & -0.50 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.489 & 0.949 & -0.46 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.422 & 0.422 \end{pmatrix}$$

G =

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 & 0 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 & 0 & 0 \\ 0 & 0 & 0 & -0.678 & 1.228 & -0.55 & 0 \\ 0 & 0 & 0 & 0 & -0.589 & 1.089 & -0.50 \\ 0 & 0 & 0 & 0 & 0 & -0.489 & 0.949 \end{pmatrix}$$

is the (n-1)×(n-1) matrix.

$$G^{-1} = \begin{pmatrix} 12.98 & 11.98 & 10.33 & 8.29 & 6.23 & 4.12 & 2.17 \\ 11.98 & 11.98 & 10.33 & 8.29 & 6.23 & 4.12 & 2.17 \\ 10.82 & 10.82 & 10.33 & 8.29 & 6.23 & 4.12 & 2.17 \\ 9.38 & 9.38 & 8.97 & 8.29 & 6.23 & 4.12 & 2.17 \\ 7.59 & 7.59 & 7.25 & 6.71 & 6.23 & 4.12 & 2.17 \\ 5.37 & 5.37 & 5.13 & 4.75 & 4.41 & 4.12 & 2.17 \\ 2.77 & 2.77 & 2.64 & 2.45 & 2.27 & 2.12 & 2.17 \end{pmatrix}$$

Rank of I = Rank of I^2 = 7

e' = (0 0 0 0 0 0 -0.422)

q' = (-1.168 -1.168 -1.114 -1.033 -0.957 -0.894 -0.915)

β = 1- q'j

= 8.249

$M' = \frac{1}{\beta}[-q', 1]$.

= (0.141 0.141 0.135 0.125 0.115 0.108 0.110 0.121)

The mean number of customers in the system is,

L = $\sum_{n=1}^{N} nM_n$

$= M_1 + 2M_2 + 3M_3 + 4M_4 + 5M_5 + 6M_6 + 7M_7$

= 3.293

The probability that the server is idle,

M_0 = 0.141

The blocking probability,

h_β = 1 - M_N

= 0.879

Model III:

$\backslash \boldsymbol{M^{(n)}} \backslash \mathbf{1/10}$

$\Delta = 0.9$

Arrival Rates (μ)	0.92	0.81	0.71	0.61	0.53	0.44	0.38	0.30	0.22	0.02
Service Rates (λ)	1	0.86	0.73	0.63	0.55	0.50	0.46	0.32	0.28	0.11

W=-I=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.678 & 1.228 & -0.55 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.589 & 1.089 & -0.50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.489 & 0.949 & -0.46 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.422 & 0.742 & -0.32 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.333 & 0.613 & -0.28 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.244 & 0.354 & -0.11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.022 & 0.022 \end{pmatrix}$$

G=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1.86 & -0.86 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.9 & 1.63 & -0.73 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.789 & 1.419 & -0.63 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.678 & 1.228 & -0.55 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.589 & 1.089 & -0.50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.489 & 0.949 & -0.46 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.422 & 0.742 & -0.32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.333 & 0.613 & -0.28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.244 & 0.354 \end{pmatrix}$$

is the $(n-1) \times (n-1)$ matrix

$$G^{-1} = \begin{pmatrix} 30.62 & 29.62 & 27.19 & 23.89 & 20.73 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 29.62 & 29.62 & 27.19 & 23.89 & 20.73 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 28.46 & 28.46 & 27.19 & 23.89 & 20.73 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 27.03 & 27.03 & 25.82 & 23.89 & 20.73 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 25.23 & 25.23 & 24.11 & 22.31 & 20.73 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 23.02 & 23.02 & 21.99 & 20.35 & 18.91 & 17.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 20.41 & 20.41 & 19.50 & 18.04 & 16.77 & 15.66 & 16.01 & 15.08 & 11.49 & 9.09 \\ 17.64 & 17.64 & 16.85 & 15.59 & 14.49 & 13.53 & 13.83 & 15.08 & 11.49 & 9.09 \\ 13.98 & 13.98 & 13.36 & 12.36 & 11.49 & 10.73 & 10.97 & 11.96 & 11.49 & 9.09 \\ 9.64 & 9.64 & 9.21 & 8.52 & 7.92 & 7.39 & 7.56 & 8.24 & 7.92 & 9.09 \end{pmatrix}$$

Rank of I = Rank of I^2 = 10

e' = (0 0 0 0 0 0 0 0 0 -0.022)

q' = (-0.212 -0.212 -0.202 -0.187 -0.174 -0.162 -0.166 -0.181 -0.174 -0.199)

β = 1- q'j

= 2.869

$M' = \frac{1}{\beta}[-q', 1]$.

=(0.073 0.073 0.070 0.065 0.060 0.056 0.057 0.063 0.060 0.069 0.348)

The mean number of customers in the system is,

L = $\sum_{n=1}^{N} nM_n$

$= M_1 + 2M_2 + 3M_3 + 4M_4 + 5M_5$

=6.292

The probability that the server is idle,

M_0 = 0.073

The blocking probability,

h_β = 1 - M_N

= 0.65

Some Important Measures:

N	**5**	**7**	**10**
L	2.787	3.293	6.292
M_0	0.220	0.141	0.073
h_β	0.831	0.879	0.65

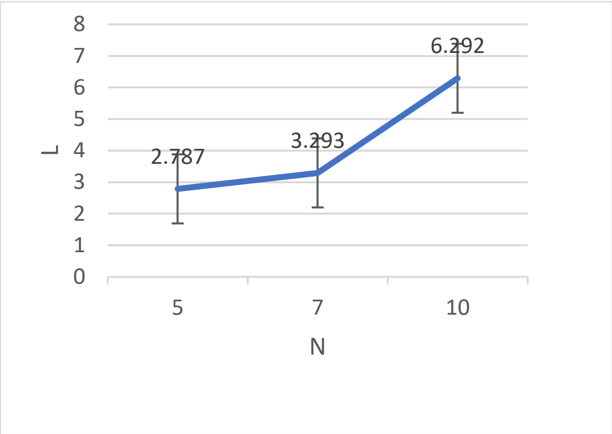

Fig: Queue Size VS the Mean Number of Customers in the System

The Number of Customers in the System will increase if the Queue Size increases.

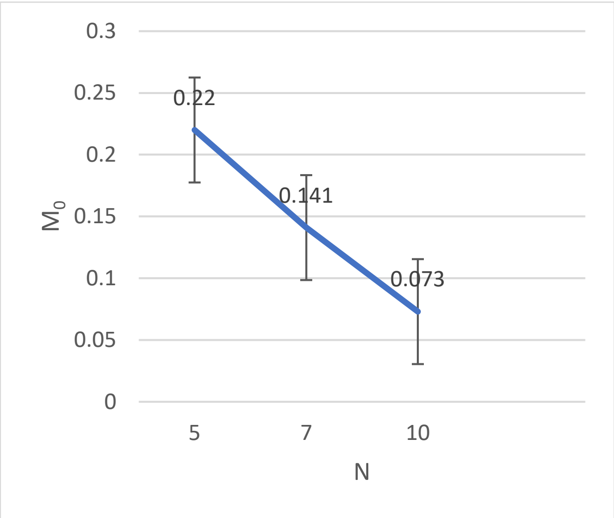

Fig: Queue size Vs idle

The idle period will decrease if the queue size increases.

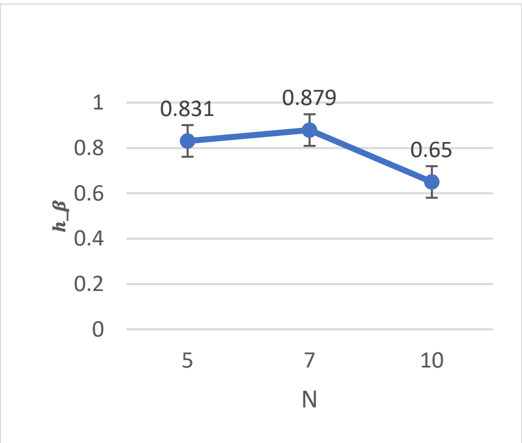

Fig: Queue size Vs Blocking Probability

The Blocking probability oscillates if the queue size increases.

Conclusion

A finite capacity single server mark ovian queue with state-dependent rate has been considered in this paper. In addition, the service rate is modified using Bernoulli probability. In this model, we analyzed the probability using group generalized inverse method. We also obtain some performance measures such as if the Queue Size increases then the mean number of customers in the system increases, idle probability decreases and blocking probability will oscillates. We also provide some numerical illustrations.

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